

CORRESPONDENCE

Comments on "On the Meridional Distribution of Source and Sink Terms of the Kinetic Energy Balance"

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The recent work of Kung (1970) on kinetic energy generation and dissipation highlights one of the most important problems relating to our understanding of the general circulation and to the construction of numerical models. In particular, when so much has been said about the geostrophic wind, gradient wind, and balance equations, it is refreshing to see the emphasis on the fact that no wind can be generated except through the $-\mathbf{V} \cdot \nabla \phi$ term expressing the cross-isobaric flow. This approach also may give useful information about frictional dissipation in the free atmosphere, a very thorny problem indeed. Although a great deal is known about frictional dissipation in the boundary layer, very little is known about dissipation in the free atmosphere. Recent views express the belief that there is considerable kinetic energy dissipation in the upper troposphere in clear-air turbulence associated with jet streams. Kung's computations give the frictional dissipation in the free atmosphere as a residual term in the kinetic energy balance equation.

Now let us consider a long-term mean closed system area that is steady state and nondivergent. Then,

$$\int_{p_0}^0 E dp = - \int_{p_0}^0 \mathbf{V} \cdot \nabla \phi dp \quad (1)$$

where E is the frictional dissipation. Also, the generation term in the boundary layer must equal the frictional dissipation in that layer. We may estimate the frictional dissipation in the Ekman layer as follows.

First, we consider the Ekman solutions for east-west oriented isobars, assuming that the surface wind is calm. Then

$$v = -\frac{1}{f} \frac{\partial \phi}{\partial y} e^{-az} \sin az \quad (2)$$

and

$$\int_0^z \rho v \frac{\partial \phi}{\partial y} dz = \rho u_g^2 \sqrt{\frac{Kf}{2}} \quad (3)$$

where K is the diffusion coefficient. Letting

$$\rho = 10^{-3} \text{ g} \cdot \text{cm}^{-3},$$

$$u_g = 10 \text{ m/s},$$

$$f = 10^{-4} \text{ s}^{-1},$$

and

$$K = 5 \times 10^4 \text{ cm}^2/\text{s},$$

TABLE 1.—Dissipation of kinetic energy by friction in the atmosphere as a function of latitude ($\text{W} \cdot \text{m}^{-2}$)

Latitude	65° 70°	60° 65°	55° 60°	50° 55°	45° 50°	40° 45°	35° 40°	30° 35°	25° 30°
Boundary layer	1.2	1.2	1.4	1.6	1.4	1.0	1.2	1.2	0.4
Free atmosphere	4.6	6.0	5.7	4.6	1.2	3.0	2.7	3.1	1.5
Total	5.8	7.2	7.1	6.2	2.6	4.0	3.9	4.3	1.9

TABLE 2.—Mean, maximum, and minimum percentage dissipation of kinetic energy in the boundary layer and free atmosphere computed from table 1

	Boundary layer	Free atmosphere
Mean	22	78
Maximum boundary	54	46
Minimum boundary	17	83

we get

$$\int_{p_0}^0 E dp = \rho u_g^2 \sqrt{\frac{Kf}{2}} = 1.58 \text{ W} \cdot \text{m}^{-2}. \quad (4)$$

Now, if we assume a reference level at anemometer height, which is the interface between the surface boundary layer and the spiral Ekman layer, we have

$$\int_{p_0}^0 E dp = \rho u_g^2 \sqrt{\frac{Kf}{2}} \sin 2\alpha \quad (5)$$

where α is the angle of intersection between the wind in the surface layer and the isobars. If $\alpha = 30^\circ$, then $\int_{p_0}^0 E dp = 1.37 \text{ W} \cdot \text{m}^{-2}$. This is a reasonable value for the dissipation of kinetic energy by friction over land in the boundary layer.

If we look at figure 8 in Kung's paper and integrate vertically, we obtain the frictional dissipation values given in table 1.

The percentages of frictional dissipation (table 2) are considerably larger for the free atmosphere than those assumed by Brunt (1926) and still quoted in the literature (Lorenz 1967).

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[Received August 16, 1971; revised August 9, 1972]

Reply^{1,2}

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The source term, $-\mathbf{V} \cdot \nabla \phi$, in the kinetic energy equation can be expressed as

$$-\mathbf{V} \cdot \nabla \phi = -\nabla \cdot \mathbf{V} \phi - \frac{\partial \omega \phi}{\partial p} - \omega \alpha.$$

While the conversion term, $-\omega \alpha$, may be regarded as the release of available potential energy, $-\nabla \cdot \mathbf{V} \phi$, and $-\partial \omega \phi / \partial p$ may be regarded as the redistribution terms required for the released energy to finally appear as the actual generation of the kinetic energy in the cross-isobaric motion.

My paper (Kung 1970), referred to by Gordon, presented a latitude-height cross-section of the $-\mathbf{V} \cdot \nabla \phi$. The distribution of $-\mathbf{V} \cdot \nabla \phi$ is markedly different from that of $-\omega \alpha$ as we generally envision it. While we have large positive $-\mathbf{V} \cdot \nabla \phi$ values to the south and north of middle latitudes in the upper troposphere, and also generally in the lower troposphere, we find a significant, negative $-\mathbf{V} \cdot \nabla \phi$ in the middle and upper troposphere of the middle latitudes. This negative area of $-\mathbf{V} \cdot \nabla \phi$ is approximately the region where we expect the maximum $-\omega \alpha$. In a subsequent paper (Kung 1971), the adiabatic generation and destruction of the kinetic energy were examined separately for the zonal and meridional motions of the atmosphere. The results show that, at the lower latitudes, kinetic energy is produced by the meridional motion and destroyed by the zonal motion; while, in the middle and higher latitudes, kinetic energy is destroyed by the meridional motion and produced by the zonal motion. It is also significant that, despite the general smallness of the meridional wind, the magnitude of $-v(\partial \phi / \partial y)$ is comparable to that of $-u(\partial \phi / \partial x)$ and plays an important role as a source term in the kinetic energy balance. These examinations of the latitude-height distribution of the source term, $-\mathbf{V} \cdot \nabla \phi$, indicate the importance of the production of the kinetic energy in Hadley cells and destruction in

Ferrel cells. The next logical step appears to be a study of the linkage between $-\omega \alpha$ and $-\mathbf{V} \cdot \nabla \phi$.

Computation of the "dissipation" term, E , as the residual term in the kinetic energy equation will remain the theoretically valid means for discussion until we become certain of the dissipation mechanisms involved. The dissipation in the planetary boundary layer usually is computed to be somewhere between 1 and 2 $\text{W} \cdot \text{m}^{-2}$, depending on the boundary layer models and climatological data employed. The values listed by Gordon obviously fall in this range. As he has indicated, the dissipation in the free atmosphere, as obtained as the residual term, is considerable and its magnitude is crucial in discussing the balance of atmospheric energy.

However, it must be stressed here that the dissipation, E , obtained as the residual term with large-scale synoptic data is nothing but the sink term of the large-scale kinetic energy balance. This is the kinetic energy removed from the grid-scale for eventual viscous dissipation. The linkage between this "sink" and eventual viscous dissipation is an open question. I do not think the planetary boundary layer can have dissipation of more than 1-2 $\text{W} \cdot \text{m}^{-2}$ —all available boundary layer models seem to predict that no more energy could be dissipated in the boundary layer with the prevailing vertical wind shear in that portion of the atmosphere. In addition, the vertical transport of kinetic energy across the top of the boundary layer is obviously negligibly small. Thus, the dissipation mechanism associated with a significant sink term in the free atmosphere shall be found in the free atmosphere. The recent studies of clear-air turbulence as discussed by Trout and Panofsky (1969), the mechanism related to cumulus convection as suggested by Gray (1970), and subgrid-scale energy analysis by McInnis and Kung (1972) may be mentioned as studies relevant in this regard. In studies of the dissipation mechanism, the kinetic energy transport by the subgrid-scale motion will be the critical point in analyzing the energy budget.

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[Received August 16, 1971; revised October 14, 1971]

¹ The research discussed in this publication was supported by the Atmospheric Science Section, National Science Foundation, under NSF Grant GA-15962.

² Contribution from the Missouri Agricultural Experiment Station, Journal Series No. 6213